Passive and Active Current Mirrors

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Overview

- Reading
  B. Razavi Chapter 5.

- Introduction
  In analog circuits current sources act as a large resistor without consuming excessive voltage headroom. This lecture deals with the design of current mirrors as both bias elements and signal processing components. Following a review of basic current mirrors, we study cascode mirror operation. Next, we analyze active current mirrors and describe the properties of differential pairs using such circuits as loads.
Basic current sources

- Application

![](image1)

- Definition of current by resistive divider

Assuming $M_1$ is in saturation, \[ I_{out} = \frac{1}{2} \mu_n C_w \frac{W}{L} \left( \frac{R_1}{R_1 + R_2} V_{DD} - V_{TH} \right)^2 \]

- The expression reveals various dependencies of $I_{out}$ upon the supply, process, and temperature.

- The overdrive voltage is a function of $V_{DD}$ and $V_{TH}$; the threshold voltage may vary by 100mV from wafer to wafer. Furthermore, both $\mu_n$ and $V_{TH}$ exhibit temperature dependence. Thus, $I_{out}$ is poorly defined.

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Basic current mirrors

- Use of a reference to generate various currents

![](image2)

A relatively complex circuit – sometimes requiring external adjustments – is used to generate a stable reference current, $I_{REF}$, which is then copied to many current sources in the system.

- Conceptual means of copying currents

How do we guarantee $I_{out} = I_{REF}$?
Basic current mirrors (cont’d)

- Basic current mirror

Diode-connected device providing inverse function.

Neglecting channel-length modulation, we can write

\[
I_{\text{REF}} = \frac{1}{2} \mu C_{\text{ox}} \left( \frac{W}{L} \right)_1 (V_{\text{GS}} - V_{\text{TH}})^2
\]

\[
I_{\text{out}} = \frac{1}{2} \mu C_{\text{ox}} \left( \frac{W}{L} \right)_2 (V_{\text{GS}} - V_{\text{TH}})^2
\]

obtaining

\[
I_{\text{out}} = \frac{(W/L)_2}{(W/L)_1} I_{\text{REF}}
\]

Basic current mirrors (cont’d)

- Currents mirrors used to bias a differential amplifier

Current mirrors usually employ the same length for all of the transistors so as to minimize error due to the side-diffusion of the source and drain areas \((L_d)\). Furthermore, the threshold voltage of short-channel devices exhibits some dependence on the channel length. Thus, current ratioing is achieving by only scaling the width of transistors.
Basic current mirrors (cont’d)

- Consider channel length modulation

\[ I_{D1} = \frac{1}{2} \mu_C C_{ox} \left( \frac{W}{L} \right)_1 (V_{GS} - V_{TH})^2 (1 + \lambda V_{DS1}) \]

\[ I_{D2} = \frac{1}{2} \mu_C C_{ox} \left( \frac{W}{L} \right)_2 (V_{GS} - V_{TH})^2 (1 + \lambda V_{DS2}) \]

obtaining

\[ \frac{I_{D1}}{I_{D2}} = \frac{(W/L)_2}{(W/L)_1} \frac{1 + \lambda V_{DS2}}{1 + \lambda V_{DS1}} \]

While \( V_{DS1} = V_{GS1} = V_{GS2} \), \( V_{DS2} \) may not equal \( V_{GS2} \) because of the circuitry fed by \( M_2 \).

Cascode current source

- Scheme – suppress the effect of channel-length modulation

How do we generate \( V_b \) in (a) to ensure \( V_Y = V_X \)?

Proper choice the dimensions of \( M_0 \) with respective to those of \( M_3 \) yields \( V_{GSO} = V_{GS3} \)

Thus, if

\[ \frac{(W/L)_3}{(W/L)_0} = \frac{(W/L)_2}{(W/L)_1} \]

then \( V_{GS3} = V_{GSO} \) and \( V_X = V_Y \).
**Cascode current source (cont’d)**

- Voltage headroom consumed by a cascode mirror

\[
V_{X} \neq V_{Y} \quad \text{(a)} \\
V_{X} = V_{Y} \quad \text{(b)}
\]

In (b), the minimum allowable voltage at node \( P \) is equal to

\[
V_{P,\text{min}} = V_{X} - V_{TH} = V_{GS0} + V_{GS1} - V_{TH}
\]

\[
= (V_{GS0} - V_{TH}) + (V_{GS1} - V_{TH}) + V_{TH}
\]

In (a), \( V_{b} \) is chosen to allow the lowest possible value of \( V_{P} \) but the output current does not accurately track \( I_{REF} \) because of \( V_{DS1} \neq V_{DS2} \). In (b), higher accuracy is achieved but the minimum level at \( P \) is higher by one threshold voltage.

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**Cascode current source (cont’d)**

- Operation of cascode current mirror
Low-voltage cascode mirror

- Modification of cascode mirror for low-voltage operation

\[ M_1, M_2 \text{ are in saturation:} \]
\[ M_2: V_b - V_{TH2} \leq V_X (= V_{GS1}) \]
\[ M_1: V_{GS1} - V_{TH1} \leq V_A (= V_b - V_{GS2}) \]
\[ \Rightarrow V_{GS2} + (V_{GS1} - V_{TH1}) \leq V_b \leq V_{GS1} + V_{TH2} \]
\[ \Rightarrow V_{GS2} - V_{TH2} \leq V_{TH1} \]

If \[ V_b = V_{GS2} + (V_{GS1} - V_{TH1}) = V_{GS4} + (V_{GS3} - V_{TH3}) \],
then the cascode current source \( M_2-M_4 \) consumes minimum headroom while \( M_1 \) and \( M_3 \) sustain equal drain-source voltages, allowing accurate copying of \( I_{REF} \).

Low-voltage cascode mirror (cont’d)

- Generation of gate voltage \( V_b \) for cascode mirror

\[ M_1, M_2 \text{ are in saturation:} \]
\[ V_{b, min} = V_{GS2} + (V_{GS1} - V_{TH1}) \]
\[ \text{Select: } V_{GS5} \approx V_{GS2}, \ V_{DS6} = V_{GS5} - RbI_1 \approx V_{GS1} - V_{TH1}. \]
\[ M_7: \text{large } (W/L) \text{, so that } V_{GST} \approx V_{TH7} \]
\[ \therefore V_{DS6} = V_{GST} - V_{TH7} \]
\[ V_b = V_{GS5} + V_{GS6} - V_{TH7} \]
Low-voltage cascode mirror (cont’d)

- Low-voltage cascode using a source follower level shifter

If $M_5$ is biased at a very low current density, $I_D/W/L$, then $V_{GS5} = V_{THS} = V_{TH3}$, i.e., $V_N' = V_N - V_{TH3}$, and

$$V_B = V_{GS1} + V_{GS0} - V_{TH3} - V_{GSS}$$

$$= V_{GSR} - V_{TH3}$$

implying that $M_2$ is at the edge of the triode region.

In this topology, however,

- $V_{DOS} \neq V_{DS1}$

- If the body effect is considered for $M_0$, $M_5$ and $M_3$, it is different to guarantee that $M_2$ operates in saturation.

Active current mirrors

- Current mirror processing a signal

$M_1$ and $M_2$ are identical: $I_{out} = I_{in}$ (for $\lambda = 0$)
- Differential pair with current source load

![Differential pair with current source load diagram]

- Calculate \( G_{in} \)

\[
G_{in} = \frac{I_{in}}{V_{in}} = \frac{g_m V_o / 2}{V_{in}} = \frac{g_m}{2} \Rightarrow |A| = G_m R_{out}
\]

- Calculate \( R_{out} \)

\[
R_{out} = (1 + g_m r_o) / \left( \frac{g_m}{2}\right) + r_o = 2r_o + \frac{g_m}{2} = 2r_o
\]

Thus, \( R_{out} \approx (2r_o) r_o \)

|A| = \frac{g_m}{2} \left(2r_o\right) r_o

- Differential pair with current source load (cont’d)

- Calculate \( V_p / V_{in} \)

\[
R_{eq} = \frac{1}{g_{m2}} + \frac{r_o}{g_{m2}} r_2 = \frac{1}{g_{m2}} \left(1 + \frac{r_o}{r_2}\right)
\]

\[
\frac{V_p}{V_{in}} = \frac{R_{eq}}{R_{eq} + \frac{1}{g_{m2}}} = 2 + \frac{r_o}{r_2}
\]

Note: if \( r_{o4} \to 0 \), \( V_p / V_{in} \to 1/2 \), and if \( r_{o4} \to \infty \), \( V_p / V_{in} \to 1 \).

- Calculate \( V_{out} / V_p \)

\[
\frac{V_{out}}{V_p} = 1 + \frac{g_m}{2} \left(\frac{r_o}{r_2}\right) + 1 + \frac{g_m}{2} \left(\frac{r_o}{r_2}\right) = \frac{g_m r_o}{2} \left(2r_o\right) r_o
\]

- Calculate \( V_{out} / V_{in} \)

\[
\frac{V_{out}}{V_{in}} = \frac{1 + \frac{r_o}{r_2}}{\frac{1}{g_{m2}} \left(1 + \frac{r_o}{r_2}\right) + \frac{1}{g_{m2}} \left(2 + \frac{r_o}{r_2}\right) r_4} = 2 + \frac{r_o}{r_2} + \frac{r_o}{r_4}
\]
Differential pair with active current mirrors

- Concept of combining the drain currents of $M_1$ and $M_2$

![Differential pair with active current mirrors diagram]

$M_3$ and $M_4$ are identical.

Differential pair with active current mirrors Large-signal analysis

- Large-signal analysis

![Differential pair with active current mirrors Large-signal analysis diagram]

Operation:
- If $V_{in1} << V_{in2}$, $M_1$ is off and so are $M_3$ and $M_4$. $M_2$ and $M_5$ operate in triode region, carrying zero current. Thus, $V_{out} = 0$.
- As $V_{in1}$ approaches $V_{in2}$ for a small difference, $M_2$ and $M_4$ are saturated, providing a high gain.
- As $V_{in1}$ becomes more positive than $V_{in2}$, $i_{D0}$, $|i_{D1}|$, and $|i_{D2}|$ increase and $i_{D2}$ decreases, eventually driving $M_4$ into the triode region.
- If $V_{in1} >> V_{in2}$, $M_2$ turns off, $M_4$ operates in deep triode region with zero current, and $V_{out} = V_{DD}$.

The choice of the input common-mode voltage:
For $M_2$ to be saturated, $V_{out} > V_{in,CM} - V_{TH}$. Thus, to allow maximum output swings, the input CM level must be as low as possible, with $V_{in,CM, min} = V_{GS1,2} + V_{DDS, min}$.
Differential pair with active current mirrors Small-signal analysis

- Small-signal analysis
  - Asymmetric swings in a differential pair with active current mirror
  - Calculate \( G_m \)

Node \( P \) can be viewed as a virtual ground.

\[
\begin{align*}
I_{D1} &= I_{D2} = \frac{g_m V_m}{2} \\
I_{D2} &= -\frac{g_m V_m}{2} \\
\Rightarrow I_{out} &= I_{D2} + I_{D4} = -g_{m,2} V_m \\
\therefore |G_m| &= g_{m,2}
\end{align*}
\]

Differential pair with active current mirrors Small-signal analysis (cont’d)

- Calculate \( R_{out} \)

\[
I_X = 2 \frac{V_x}{2r_{n,2}} + \frac{V_x}{r_{n,3}}
\]

where the factor 2 accounts for current copying action of \( M_3 \) and \( M_4 \).

For \( 2r_{n,2} \gg (1/g_m) r_{n,3} \), we have \( R_{out} = r_{o2} \parallel r_{o4} \)

- Calculate \( A_v \)
  \[
  |A_v| = |G_m R_{out}| = g_{m,2} (r_{o2} \parallel r_{o4})
  \]
Differential pair with active current mirrors Small-signal analysis (cont’d)

- Substitution of the input differential pair by a Thevenin equivalent

\[ V_{eq} = g_{m1,2} r_{o1,2} V_{in} \]
\[ R_{eq} = 2 r_{o1,2} \]

- Calculate \( V_{eq} \) and \( R_{eq} \)

\[ V_{eq} = \frac{g_{m1,2}}{r_{o1,2}} \cdot V_{in} \]
\[ R_{eq} = 2 r_{o1,2} \]

Differential pair with active current mirrors Small-signal analysis (cont’d)

- Calculate \( A_v = \frac{V_{out}}{V_{in}} \)

The current through \( R_{eq} \) is

\[ I_{X1} = \frac{V_{out} - g_{m1,2} r_{o1,2} V_{in}}{2 r_{o1,2} + \frac{1}{g_{m3}}} \]

The fraction of this current that flows through

\[ \frac{1}{g_{m3}} \] is mirrored into \( M_4 \) with unity gain. That is,

\[ I_{X1} \cdot \frac{V_{out} - g_{m1,2} r_{o1,2} V_{in}}{2 r_{o1,2} + \frac{1}{g_{m3}}} \cdot \frac{1}{g_{m3}} + \frac{V_{out}}{r_{o4}} = 0 \]

Assuming \( 2 r_{o1,2} \gg \frac{1}{g_{m3}} \), we obtain

\[ \frac{V_{out}}{V_{in}} = g_{m1,2} r_{o1,2} r_{o3,4} = \frac{g_{m1,2} r_{o1,2}}{r_{o1,2} + r_{o3,4}} \]
Differential pair with active current mirrors Common-mode properties

- Differential pair with active current mirror sensing a common-mode change

![Circuit Diagram]

The CM gain is defined in terms of the single-ended output component produced by the input CM change:

\[ A_{CM} = \frac{\Delta V_{out}}{\Delta V_{in,CM}} \]

Differential pair with active current mirrors Common-mode properties (cont’d)

- Simplified circuit of CM circuit

\[ A_{CM} = \frac{1}{2g_{m3,4}} \frac{g_{m1,2}}{2} + \frac{2g_{m1,2} + R_{o3,4}}{2} \]

\[ = \frac{1}{1 + 2g_{m1,2}R_{o3,4}} \frac{g_{m1,2}}{g_{m3,4}} \]

where we have assumed \(1/(2g_{m3,4}) \ll r_{o3,4}\), and neglected the effect of \(r_{o1,2}/2\).

Even with perfect symmetry, the output signal is corrupted by input CM variations, a drawback that does not exist in the fully differential circuits.

- CMRR

\[ CMRR = \frac{A_{out}}{A_{CM}} = g_{m1,2} \frac{r_{o1,2} / / r_{o3,4}}{g_{m3,4}} \frac{g_{m1,2} (1 + 2g_{m1,2}R_{o3,4})}{g_{m1,2}} = (1 + 2g_{m1,2}R_{o3,4})g_{m3,4} (r_{o1,2} / / r_{o3,4}) \]
Differential pair with active current mirrors Mismatch

- Differential pair with \( g_m \) mismatch

Considering \( M_1 \) and \( M_2 \) as a single transistor with \( g_m = g_{m1} + g_{m2} \),

\[
\Delta V_p = \Delta V_{m,CM} \frac{R_{SS}}{R_{SS} + \frac{1}{g_{m1} + g_{m2}}} 
\]

where body effect is neglected. The change of \( I_{D1} \) and \( I_{D2} \) are given by

\[
\Delta I_{D1} = g_m (\Delta V_{m,CM} - \Delta V_p) = \frac{\Delta V_{m,CM}}{R_{SS} + \frac{1}{g_{m1} + g_{m2}}} g_{m1} 
\]

\[
\Delta I_{D2} = g_m (\Delta V_{m,CM} - \Delta V_p) = \frac{\Delta V_{m,CM}}{R_{SS} + \frac{1}{g_{m1} + g_{m2}}} g_{m2} 
\]

Differential pair with active current mirrors Mismatch (cont’d)

And \( |\Delta I_{D4}| = g_{m4} |\Delta V_{D4}| = g_{m4} \left( \frac{1}{r_5} / \frac{1}{r_5} \right) \Delta I_{D1} \)

Neglecting the effect of \( r_{o1} \) and \( r_{o2} \):

\[
\Delta V_{out} = (\Delta I_{D4} - \Delta I_{D2})/r_4 
\]

\[
= \left[ \frac{g_m \Delta V_{m,CM}}{1 + (g_{m1} + g_{m2}) R_{SS}} \frac{r_{o1}}{r_3} + \frac{1}{r_3} \frac{g_m \Delta V_{m,CM}}{1 + (g_{m1} + g_{m2}) R_{SS}} \right] r_4 
\]

\[
= \frac{\Delta V_{m,CM}}{1 + (g_{m1} + g_{m2}) R_{SS}} (g_{m1} - g_{m2}) R_{SS} - g_{m3} / g_{m3} r_4 
\]

If \( r_3 \gg 1/g_{m3} \), we have

\[
\frac{\Delta V_{out}}{\Delta V_{m,CM}} = \frac{(g_{m1} - g_{m2}) r_3 - g_{m2} / g_{m3}}{1 + (g_{m1} + g_{m2}) R_{SS}} 
\]

\((g_{m1} - g_{m2}) r_3 \) reveals the effect of the transconductance Mismatch on the common-mode gain.
Supply-independent biasing

- Current-mirror biasing using (a) an ideal current source, (b) a resistor.

To output current is quite sensitive to $V_{DD}$:

$$I_{out} = \frac{\Delta V_{DD}}{R_1 + 1/g_{m2}} \left( \frac{W}{L} \right)_2$$

How do we generate $I_{REF}$ independent of the supply voltage?

- Simple circuit to establish supply-independent currents.

In order to arrive at a less sensitive solution, we postulate that the circuit must bias itself, i.e., $I_{REF}$ must be somehow derived from $I_{out}$.

If $M_1 \cdot M_4$ operate in saturation and $\lambda = 0$, then $I_{out} = K I_{REF}$, and hence can support any current level.

Supply-independent biasing

- Addition of $R_S$ to define the currents

Assuming $\lambda = 0$, then $I_{out} = I_{REF}$ and $V_{G81} = V_{G82} + I_{D2} R_S$

Neglecting body effect, we have

$$\frac{2I_{out}}{\mu_n C_{ox}(W/L)_N} + V_{m1} = \frac{2I_{out}}{\mu_n C_{ox}(K(W/L)_N)} + V_{m2} + I_{out} R_S$$

That is, $I_{out} = \frac{2}{\mu_n C_{ox}(W/L)_N} \frac{1}{R_S} \left( 1 - \frac{1}{\sqrt{K}} \right)^2$ The current is independent of the supply voltage (but still a function of process and temperature).
Supply-independent biasing (cont’d)

- Addition of $R_s$ to define the currents (assuming $\lambda \neq 0$). Determine $\Delta I_{out}/\Delta V_{DD}$.

$$R_1 = r_{o1} \parallel (1/g_m1), \quad R_3 = r_{o3} \parallel (1/g_m3)$$

$$\frac{V_{DD} - V_A}{r_{o1}} + I_{out} R_s g_m = \frac{V_A}{R_s}$$

The equivalent transconductance of $M_2$ and $R_s$ is $g_{m2} = \frac{I_{out}}{V_A} = \frac{g_m2 r_{o2}}{R_s + r_{o2} + (g_{m2} + g_{m3}) R_s r_{o2}}$

Thus, $\frac{I_{out}}{V_{DD}} = \frac{1}{r_{o4} \left[ \frac{1}{G_{m2}(r_{o2}/R_s) - g_m3 R_s} \right]} \rightarrow 0$, if $r_{o4} = \infty$.

Supply-independent biasing (cont’d)

- Addition of $R_s$ to define the currents

- An important issue in supply-independent biasing is the existence of “degenerate” bias points. For example, if all the transistors carry zero current when the supply is turned on, they may remain off indefinitely because the loop can support a zero current in both branches.

- In other words, the circuit can settle in one of two different operating conditions.

- Addition of start-up device

- The diode-connected device $M_6$ provides a current path from $V_{DD}$ through $M_6$ and $M_7$ to ground upon start-up.

- This technique is practical if $V_{TH1} + V_{TH5} + |V_{TH3}| < V_{DD}$ and $V_{GSL} + V_{TH5} + |V_{GSL}| > V_{DD}$, the latter to ensure $M_6$ remains off after start-up.