Phase-Locked Loops

Charge-Pump Phase-Locked Loops

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Conceptual operation of a phase-frequency detector (PFD)

\[ \phi_A \neq \phi_B \]

\[ \omega_A \neq \omega_B \]
Phase detector : PFD – three-state PD

State diagram

Timing diagram

PLL ICs
- Implementation of PFD

Input-output characteristic:

PFD followed by low-pass filters:

PLL ICs
- Phase detector : PFD

- When locked, the phase difference is 0 degree
- Output voltage *independent* on the input signal amplitudes
- Output voltage *independent* on the input duty cycles
- Wide linear range (Wide lock range) $\Rightarrow \pm 2\pi$
- Discriminate frequency difference
- Use carefully for Data/Clock Recovery PLL
  - Hybrid PLL (Analog PLL + Digital PLL)
    - *Dead Zone problem*
      - *Due to finite gate delay*
      - *Introduce large jitter or poor phase noise*
Charge-pump PLL

Why charge pump PLL?

- **Advantages**
  - No active component for zero steady state phase error
  - Large frequency and phase capture range
  - Digital output (full CMOS swing)
  - Simple and robust design
  - Discrete time analysis

- **Disadvantages**
  - Slow comparing with analog PLL
  - May create dead zone problem
  - Noisy
Addition of zero to charge-pump PLL (2\textsuperscript{nd} order)

- **Open-loop transfer function**
  \[
  \phi_{\text{out}}(s)
  \bigg|_{\text{open}} = \frac{I_P}{2\pi} \left( \frac{1}{R_1 + \frac{1}{C_1s}} \right) \frac{K_{VCO}}{s} \quad \Rightarrow \text{a zero at } s_z = -\frac{1}{(R_1C_1)}.
  \]

- **Closed-loop transfer function**
  \[
  H(s) = \frac{\frac{I_P K_{VCO}}{2\pi C_1} (R_1C_1s + 1)}{s^2 + \frac{I_P}{2\pi} K_{VCO} R_1 s + \frac{I_P}{2\pi C_1} K_{VCO}}
  \]

\[
\Rightarrow \quad \omega_n = \sqrt{\frac{I_P K_{VCO}}{2\pi C_1}}, \quad \zeta = \frac{R_1}{2} \sqrt{\frac{I_P C_1 K_{VCO}}{2\pi}} \quad \text{and decay time constant} \quad \frac{1}{\zeta \omega_n} = \frac{4\pi}{R_1 I_1 K_{VCO}}
\]
Critical drawback:

- Since the charge pump drives the series combination of $R_P$ and $C_P$, each time a current is injected into the loop filter, the control voltage experiences a large jump.
- In lock condition, the mismatches between $I_1$ and $I_2$ and charge injection and clock feedthrough of $S_1$ and $S_2$ introduce voltage jumps in $V_{cont}$.

⇒ The resulting ripple severely disturbs the VCO, corrupting the output phase.
PLL frequency synthesizer using dividers

In the locked state:

\[ F_{\text{VCO}} = N \times F_{\text{R}}, \quad N \in \mathbb{N} \]

\[ F_{\text{VCO}} / N = F_{X} / R = F_{\text{R}} \]
Linear model of 3\textsuperscript{rd}-order PLLs with 2\textsuperscript{nd}-order loop filter

PLL phase transfer functions:

**Forward-loop gain**

\[ G(s) = \frac{\theta_o}{\theta_e} = \frac{K_{PD}Z_{LF}(s)K_{vco}}{s} \]

**Reverse-loop gain**

\[ \beta(s) = \frac{\theta_{div}}{\theta_o} = \frac{1}{N} \]

**Open-loop gain**

\[ \beta(s)G(s) = \frac{\theta_{div}}{\theta_e} = \frac{K_{PD}Z_{LF}(s)K_{vco}}{Ns} \]

**Closed-loop gain**

\[ \frac{\theta_o}{\theta_{ref}} = \frac{G(s)}{1 + \beta(s)G(s)} \]
- 2\textsuperscript{nd}-order passive filter

(Addition of $C_2$ to reduce ripple on the control line.)

$$Z_{LF}(s) = \frac{k \cdot \frac{1 + s \tau_z}{1 + s \tau_p}}{\frac{1 + s \tau_z}{b}} \quad \text{or} \quad \frac{k \cdot \frac{1 + s \tau_z}{1}}{\frac{1 + s \tau_z}{b}}$$

$$Z_{LF1}(s) = \frac{1 + s R_1 C_1}{s^2 R_1 C_1 C_2 + s (C_1 + C_2)} = \frac{1 + s \tau_z}{s (C_1 + C_2)(1 + s \tau_p)}$$

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Loop Filter 1</th>
<th>Loop Filter 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_z$</td>
<td>$R_1 C_1$</td>
<td>$R_1 (C_1 + C_2)$</td>
</tr>
<tr>
<td>$\tau_p$</td>
<td>$R_1 \left(\frac{C_1 C_2}{C_1 + C_2}\right)$</td>
<td>$R_1 C_2$</td>
</tr>
<tr>
<td>$k$</td>
<td>$\left(\frac{b-1}{b}\right) \frac{1}{C_1}$</td>
<td>$\frac{1}{C_1}$</td>
</tr>
<tr>
<td>$b = \frac{\tau_z}{\tau_p}$</td>
<td>$1 + \frac{C_1}{C_2}$</td>
<td>$1 + \frac{C_1}{C_2}$</td>
</tr>
</tbody>
</table>

$$Z_{LF2}(s) = \frac{1 + s R_1 (C_1 + C_2)}{s C_1 (1 + s R_1 C_2)}$$
- Open-loop bandwidth $\omega_c$ and phase margin $\phi_m$

Open-loop gain

$$\beta(s)G(s) = \frac{\theta_{\text{div}}}{\theta_e} = \frac{K_{PD}Z_{LF}(s)K_{vco}}{Ns} = \frac{I_P \cdot K_{vco} \cdot k}{2\pi \cdot N \cdot s^2} \cdot \frac{1 + s \tau_z}{1 + s \tau_p}$$

$$\Rightarrow \beta(s)G(s)\big|_{s=j\omega} = -\frac{I_P \cdot K_{vco} \cdot k}{2\pi \cdot N \cdot \omega^2} \cdot \frac{1 + j \omega \tau_z}{1 + j \omega \tau_p}$$

$$\phi(\omega) = \tan^{-1}(\omega \cdot \tau_z) - \tan^{-1}(\omega \cdot \tau_p) - 180^\circ$$

Bode plot of open loop response

$$\phi_{m,\text{max}}: \frac{d\phi}{d\omega} = \frac{\tau_z}{1 + (\omega \cdot \tau_z)^2} - \frac{\tau_p}{1 + (\omega \cdot \tau_p)^2} = 0$$

$$\Rightarrow \omega_c = \frac{1}{\sqrt{\tau_z \cdot \tau_p}}$$

and $\phi_{m,\text{max}} = \tan^{-1}\left(\frac{\tau_z - \tau_p}{2\sqrt{\tau_z \cdot \tau_p}}\right) = \tan^{-1}\left(\frac{b - 1}{2\sqrt{b}}\right)$
If the loop bandwidth $\omega_c$ and the phase margin $\phi_m$ are specified, we have

$$b = \frac{1}{\left(-\tan\phi_m + \frac{1}{\cos\phi_m}\right)^2}$$

and

$$\tau_z = \frac{\sqrt{b}}{\omega_c} \quad \tau_p = \frac{1}{\sqrt{b \cdot \omega_c}}$$

For loop filter 1:

$$\omega_c = \frac{I_p K_{vco}}{2\pi \cdot N} R_1 \frac{b - 1}{b} = \frac{I_p K_{vco}}{2\pi \cdot N} R_1 \frac{C_1}{C_1 + C_2}$$

$$R_1 = \frac{2\pi \cdot N \cdot \omega_c}{I_p \cdot K_{vco}} \frac{b}{b - 1}$$

$$C_1 = \frac{\tau_z}{R_1} \quad \text{and} \quad C_2 = \frac{1}{R_1} \cdot \frac{\tau_z \tau_p}{\tau_z - \tau_p}$$

For loop filter 2:

$$\omega_c = \frac{I_p K_{vco}}{2\pi \cdot N} R_1 \frac{b}{b - 1} = \frac{I_p K_{vco}}{2\pi \cdot N} R_1 \frac{C_1 + C_2}{C_1}$$

$$R_1 = \frac{2\pi \cdot N \cdot \omega_c}{I_p \cdot K_{vco}} \frac{b - 1}{b}$$

$$C_1 = \frac{\tau_z - \tau_p}{R_1} \quad \text{and} \quad C_2 = \frac{\tau_p}{R_1}$$
Active loop filter implementation

The active loop filter is often used when the charge-pump output cannot directly provide the required voltage range for tuning of the VCO. Such voltages are incompatible with charge-pumps built in standard IC technologies, so that a (partly external) active loop filter is then used to isolate the charge-pump output from the VCO tuning input, and to generate the high tuning voltages.
Design flow of 3\textsuperscript{rd}-order PLLs

\begin{enumerate}
\item Determine $K_{vco}$.
\item Determine the nominal value of $N$ according to the system to be applied to.
\item Depending on the desired noise and transient performance, determine the loop bandwidth $\omega_c$.
\item Select $I_p$, to meet the reasonable trade-off between the value the filter components (i.e., chip area) and the pump current.
\item Select the required phase margin $\phi_m$ or $b$.
\item With $K_{vco}$, $N$, $\omega_c$, $I_p$ and $b$ determined, calculate $R_1$.
\item With $\tau_z$ and $\tau_p$ determine by $\omega_c$ and $b$, calculate $C_1$ and $C_2$.
\end{enumerate}
Multi-path charge-pump filter (1/3)

The loop filter transfer function is

\[ \frac{V_{\text{cont}}(s)}{\theta_e(s)} = \frac{K_{pd} Z_{LF}(s)}{I_{p1}} = \frac{1}{2\pi sC_a} + \frac{1}{2\pi} \left( R_b \frac{1}{sC_b} \right) \]

\[ = \frac{I_{p1}}{2\pi \cdot sC_a} \cdot \frac{sC_b \left( C_b + C_a \frac{I_{p2}}{I_{p1}} \right) + 1}{sR_b C_b + 1} \]

\[ \Rightarrow \quad \frac{1}{\omega_z} = R_b \left( C_b + C_a \frac{I_{p2}}{I_{p1}} \right) \approx R_b C_a \frac{I_{p2}}{I_{p1}} \]

\[ \frac{1}{\omega_p} = R_b C_b \]

\[ \omega_c \approx \frac{I_{p1} \cdot K_{vco}}{2\pi \cdot N \cdot C_a \cdot \omega_z} = \frac{K_{vco}}{2\pi \cdot N} \cdot I_{p2} \cdot R_b \]

The capacitor \( C_a \) can be multiplied by \( n = I_{p2} / I_{p1} (> 1) \).


PLL ICs

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Multi-path charge-pump filter (2/3)

The filter transfer function is

\[ F(s) = \frac{V_{\text{cont}}}{i_e} = \frac{sR_b(nC_a + C_b) + 1}{s^2nR_bC_aC_b + snC_a} \]

\[ \Rightarrow \quad \frac{1}{\omega_c} = nR_b(C_b + nC_a) \approx nR_bC_a \quad \frac{1}{\omega_p} = R_bC_b \]

The open-loop transfer function is

\[ G(s) = \frac{I_{p2} \cdot K_f \cdot K_{\text{VCO}}}{2\pi \cdot N} \cdot \frac{s + \omega_c}{s^2 \left( \frac{s}{\omega_p} + 1 \right)} \]

where \[ K_f = \left( 1 + \frac{C_b}{nC_a} \right) R_b \]

\[ \Rightarrow \quad \omega_c \approx \left( 1 + \frac{C_b}{nC_a} \right) \frac{I_{p2} \cdot R_b \cdot K_{\text{VCO}}}{2\pi \cdot N} \]

\[ \omega_c \text{ is larger than the traditional one.} \]
Multi-path charge-pump filter (3/3)

The open-loop transfer function is

\[ G(s) = \frac{I_{p1} \cdot K_f \cdot K_{VCO}}{2\pi \cdot N} \cdot \frac{s + \omega_c}{s^2 \left( \frac{s}{\omega_p} + 1 \right)} \]

where \( K_f = \left( n + \frac{C_b}{C_a} \right) R_b \)

\( \Rightarrow \omega_c \approx \left( 1 + \frac{C_b}{nC_a} \right) \frac{nI_{p1} \cdot R_b \cdot K_{VCO}}{2\pi \cdot N} \)

\( \omega_c \) is larger than the traditional one.

The filter transfer function is

\[ F(s) = \frac{V_{cont}}{i_{e1}} = \frac{sR_b \left( nC_a + C_b \right) + 1}{s^2 nR_b C_a C_b + snC_a} \]

\( \Rightarrow \quad \frac{1}{\omega_p} = R_b \left( C_b + nC_a \right) \approx nR_b C_a \quad \frac{1}{\omega_p} = R_b C_b \)

The closed-loop transfer function is

\[ H(s) = \frac{G(s)N}{1 + G(s)} = \frac{I_{p1} K_{VCO}}{2\pi C_a} \left( nR_b C_a s + 1 \right) \]

\( \Rightarrow \quad \omega_n = \sqrt{\frac{I_{p1} K_{VCO}}{2\pi C_a}} \quad \zeta = \frac{nR_b}{2} \sqrt{\frac{I_{p1} K_{VCO}}{2\pi N}} \)
Nonideal Effects in PLLs
Owing to the finite risetime and falltime resulting from the capacitance seen at the nodes, the pulse may not find enough time to reach a logical high level, failing to turn on the charge pump switches.

- For $|\Delta \phi| < \phi_0$, the charge pump injects no current.
- The loop gain drops to zero and the output phase is not locked.
- The PFD/CP suffers from a dead zone equal to $\pm \phi_0$ around $\Delta \phi = 0$.

$\Rightarrow$ Jitter resulting from the dead zone
Reference spurs

- Periodic disturbance of VCO control line due to charge pump activity:

\[ i_e(t) = I_P \delta_{cp} + 2I_P \delta_{cp} \sum_{n=1}^{\infty} \cos(2\pi nf_{ref} t) \]

- DC 
- Spectral components

- Main effects which generate reference spurious breakthrough:
  - leakage current in the loop filter,
  - skew between up and down (dn) signals,
  - mismatch in the charge up and down current sources,
  - Charge sharing.
- Effect of leakage current

- Sources of leakage currents:
  - the capacitor of loop filter,
  - the input of VCO,
  - the charge-pump output,
  - the input biasing current of the op-amp, when active loop filter configuration is used.

- The duty cycle of the charge-pump output is

\[
\bar{i}_e = I_P \delta_{cp} = I_{\text{leak}} \quad \Rightarrow \quad \delta_{cp} = \frac{I_{\text{leak}}}{I_P}
\]

- The amplitude of charge-pump output:

\[
i_e(t) = I_{\text{leak}} + 2I_{\text{leak}} \sum_{n=1}^{\infty} \cos(2\pi nf_{\text{ref}} t)
\]

1. The spectral component are twice the value of \( I_{\text{leak}} \).
2. Not dependent on the nominal charge-pump current \( I_P \).
- Link the leakage current to the magnitude of the spurious components at the output of the VCO:

\[ V_{\text{ripple}}(n \cdot f_{\text{ref}}) = 2I_{\text{leak}} |Z_{\text{LF}}(j2\pi nf_{\text{ref}})| \]

- Phase deviation

\[ \theta_p(n \cdot f_{\text{ref}}) = \frac{\Delta f(n \cdot f_{\text{ref}})}{n \cdot f_{\text{ref}}} = \frac{V_{\text{ripple}}(n \cdot f_{\text{ref}})K_{\text{vco}}}{n \cdot f_{\text{ref}}} = \frac{2I_{\text{leak}} |Z_{\text{LF}}(j2\pi nf_{\text{ref}})| K_{\text{vco}}}{n \cdot f_{\text{ref}}} \]

1. Each of baseband modulation frequencies \(n \cdot f_{\text{ref}}\) generates two RF spurious signals at offset frequencies \(\pm n \cdot f_{\text{ref}}\) from the carrier \(f_{\text{LO}}\).
2. The amplitude of each spurious signal

\[ A_{\text{SP}}(f_{\text{LO}} \pm n \cdot f_{\text{ref}}) = A_{\text{LO}} \frac{\theta_p(n \cdot f_{\text{ref}})}{2} = A_{\text{LO}} \frac{I_{\text{leak}} |Z_{\text{LF}}(j2\pi nf_{\text{ref}})| K_{\text{vco}}}{n \cdot f_{\text{ref}}} \]

\[ \Rightarrow \frac{A_{\text{SP}}(f_{\text{LO}} \pm n \cdot f_{\text{ref}})}{A_{\text{LO}}} = \frac{I_{\text{leak}} |Z_{\text{LF}}(j2\pi nf_{\text{ref}})| K_{\text{vco}}}{n \cdot f_{\text{ref}}} \]

\[ \Rightarrow \left[ \frac{A_{\text{SP}}}{A_{\text{LO}}} \right]_{\text{dBc}} = 20 \log \left( \frac{\theta(n \cdot f_{\text{ref}})}{2} \right) = 20 \log \left( \frac{I_{\text{leak}} |Z_{\text{LF}}(j2\pi nf_{\text{ref}})| K_{\text{vco}}}{n \cdot f_{\text{ref}}} \right) [\text{dBc}] \]

The relative amplitude of the spurious signal is *not* dependent on the value of loop bandwidth or on the nominal charge-pump current \(I_P\).

Theoretically, if \(I_{\text{leak}} = 0\) there are no reference spurs in the output.
Effect of skew between *up* and *down* signals
- Effect of mismatch in the charge-pump current sources

**PFD**

\[ V_{DD} \quad V_{ref} \quad V_{div} \quad V_{DD} \]

\[ D \quad Q \quad \text{rst} \quad D \quad Q \]

\[ \text{skew suppression} \]

\[ V_{op} \quad V_{b1} \quad V_{b2} \]

\[ I_{up} \quad I_{dn} \quad I_e \quad I_o \]

\[ C \]

\[ V_{cont} \]

\[ t \]
- **Effect of mismatch in the charge-pump current sources**

- For the loop to remain locked, the average value of $V_{\text{cont}}$ must remain constant. The PLL therefore creates a phase error between the input and the output such that the net current injected by the charge pump in every cycle is zero.
  - The control voltage still experiences a periodic ripple.
  - Owing to the low output impedance of short-channel MOSFETs, the current mismatch varies with the output voltage.
  - The clock feedthrough and charge injection mismatch between $M_1$ and $M_2$ further increases both the phase error and the ripple.

- The magnitude of the spectral components of the ripple voltage due to current-source mismatch can be found

  $$V_{\text{mismatch}}(n \cdot f_{\text{ref}}) = I_{\text{out}}(n \cdot f_{\text{ref}}) \cdot |Z_{\text{LF}}(j2\pi nf_{\text{ref}})|$$

  $\Rightarrow$  

  $$\left[ \frac{A_{sp}(n \cdot f_{\text{ref}})}{A_{\text{LO}}} \right]_{\text{dBc}} = 20\log \frac{I_{\text{out}}(n \cdot f_{\text{ref}}) |Z_{\text{LF}}(j2\pi nf_{\text{ref}})| K_{\text{vco}}}{2 \cdot n \cdot f_{\text{ref}}}$$
- Effect of charge sharing

Charge sharing between $C_P$ and capacitances at $X$ and $Y$:

- $S_1$ and $S_2$ are off, allowing $M_3$ to discharge $X$ to ground and $M_4$ to charge $Y$ to $V_{DD}$.
- At the next phase comparison instant, both $S_1$ and $S_2$ turn on, $V_X$ rises, $V_Y$ falls, and $V_X \approx V_Y \approx V_{cont}$.

- If the phase error is zero and $I_{D3} = |I_{D3}|$, does $V_{cont}$ remain constant after the switches turn on? Even if $C_X = C_Y$, the change in $V_X$ is not equal to that in $V_Y$. 

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- Effect of charge sharing

- Bootstrapping X and Y to minimize charge sharing:
  - When $S_1$ and $S_2$ turn off, $S_3$ and $S_4$ turn on, allowing the unity-gain amplifier to hold nodes X and Y at a potential equal to $V_{cont}$.
  - At the next phase comparison instant, $S_1$ and $S_2$ turn on, $S_3$ and $S_4$ turn off, and $V_X$ and $V_Y$ begin with a value equal to $V_{cont}$.

- The ideal is to “pin” $V_X$ and $V_Y$ to $V_{cont}$ after phase comparison is finished. Thus, no charge sharing occurs between $C_P$ and the capacitances at X and Y.