Microelectronic Circuits

Feedback

Ching-Yuan Yang

National Chung-Hsing University
Department of Electrical Engineering

Outline

- The General Feedback Structure
- Some Properties of Negative Feedback
- The Four Basic Feedback Topologies
- The Series-Shunt Feedback Amplifier
- The Series-Series Feedback Amplifier
- The Shunt-Shunt and Shunt-Series Feedback Amplifier
- Determining the Loop Gain
- The Stability Problem
- Effect of Feedback on the Amplifier Poles
- Stability study Using Bode Plots
- Frequency Compensation
The General Feedback Structure

General structure of the feedback amplifier

- \( x_o = Ax_i \), \( x_f = \beta x_o \), \( x_i = x_s - x_f \)  \( \Rightarrow \) \( A_f \equiv \frac{x_o}{x_s} = \frac{A}{1 + A\beta} \)

- \( \beta \): feedback factor
- \( A\beta \): loop gain
- \( 1 + A\beta \): amount of feedback
- If \( A\beta >> 1 \), then \( A_f \approx \frac{1}{\beta} \).
- The gain of the feedback amplifier is almost entirely determined by the feedback network.
- Feedback signal: \( x_f = \frac{A\beta}{1 + A\beta} x_s \)

For \( A\beta >> 1 \) we see that \( x_f \approx x_s \), which implies that \( x_i \to 0 \). (\( x_i \): error signal)
Some Properties of Negative Feedback

- Desensitize the gain
- Extend the bandwidth of the amplifier
- Reduce the effect of noise
- Reduce nonlinear distortion
Gain Desensitivity

Assume that \( \beta \) is constant.

\[
A_f \equiv \frac{x_o}{x_s} = \frac{A}{1 + A\beta}
\]

\[
\Rightarrow \quad \frac{dA_f}{dA} = \frac{1}{(1 + A\beta)^2}
\]

\[
\Rightarrow \quad dA_f = \frac{dA}{(1 + A\beta)^2}
\]

\[
\Rightarrow \quad \frac{dA_f}{A_f} = \frac{1}{1 + A\beta} \frac{dA}{A}
\]

The percentage change in \( A_f \) (due to variations in some circuit parameter) is smaller than the percentage change in \( A \) by amount of feedback.

The amount of feedback, \( 1 + A\beta \), is also known as the desensitivity factor.

Bandwidth Extension

Consider an amplifier whose high-frequency response is characterized by a single pole.

\[
A(s) = \frac{A_M}{1 + \frac{s}{\omega_H}} \quad A_M: \text{the midband gain} \quad \omega_H: \text{the upper 3-dB frequency}
\]

Apply the negative feedback with a frequency-independent factor \( \beta \):

\[
\text{Close-loop gain} \quad A_f(s) = \frac{A(s)}{1 + \beta A(s)} = \frac{A_M}{1 + A_M \beta} \frac{1}{1 + \frac{s}{\omega_H(1 + A_M \beta)}}
\]

The feedback amplifier: (high-frequency response)

1. Midband gain \( A_{Mf} = \frac{A_M}{1 + A_M \beta} \)

2. Upper 3-dB frequency \( \omega_{Hf} = \omega_H(1 + A_M \beta) \)

- The amplifier bandwidth is increased by the same factor, \( 1 + A_M \beta \), by which its midband gain is decreased, maintain the gain-bandwidth product constant.

- Similarly, the low-frequency response of the feedback amplifier has a lower 3-dB frequency

\[
\omega_{lf} = \frac{\omega_l}{1 + A_M \beta}
\]
### Noise Reduction

The amplifier suffers from noise and the noise can be assumed to be introduced at the input of the amplifier.

Negative feedback is applied to improve SNR:

\[
V_o = V_s \frac{A_1 A_2}{1 + A_1 A_2 \beta} + V_n \frac{A_1}{1 + A_1 A_2 \beta}
\]

\[
\therefore \frac{S}{N} = V_o = V_s A_2
\]

### Reduction in Nonlinear Distortion

The open-loop voltage transfer characteristic of the amplifier is changed from 1000 to 100 and then to 0. [curve (a)]

Apply negative feedback with \( \beta = 0.01 \) to the amplifier [curve (b)]:

1. the slope of the steepest segment:
   \[
   A_{f1} = \frac{1000}{1 + 1000 \times 0.01} = 90.9
   \]

2. the slope of the next segment:
   \[
   A_{f2} = \frac{100}{1 + 100 \times 0.01} = 50
   \]

\( \Rightarrow \) The order-of-magnitude change in slope is considerably reduced.

\( \Rightarrow \) The price paid is a reduction in voltage gain.
The Four Basic Feedback Topologies

Four basic feedback topologies

- Base on the quantity to be amplified (voltage or current) and on the desired form of the output (voltage or current), amplifiers can be classified into four categories.
  - Voltage amplifiers
  - Current amplifiers
  - Transconductance amplifiers
  - Transresistance amplifiers
- Four basic feedback topologies:
  - Voltage-sampling series-mixing (series-shunt) topology
  - Current-sampling shunt-mixing (shunt-series) topology
  - Current-sampling series-mixing (series-series) topology
  - Voltage-sampling shunt-mixing (shunt-shunt) topology
Voltage Amplifiers

- Characteristics:
  - Input signal: voltage  output signal: voltage
  - Voltage-controlled voltage source
  - High input impedance
  - Low output impedance

- Feedback topology
  - Voltage-sampling series-mixing (series-shunt) topology
    - The feedback network samples the output voltage, and the feedback signal \( x_f \) is a voltage that can be mixed with the source voltage in series.
    - “Series” refers to the connection at the input and “shunt” refers to the connection at the output.

  - Characteristics:
    - Stabilize the voltage gain
    - Higher input resistance
      (series connection at the input)
    - Lower output resistance
      (parallel connection at the output)

\[ v_i = V_+ - V_- \]

- Example
Current Amplifiers

- Characteristics:
  - Input signal: current  output signal: current
  - Current-controlled current source
  - Low input impedance
  - High output impedance

- Feedback topology
  - Current-sampling shunt-mixing (shunt-series) topology
    - The feedback network *samples* the output current, and the feedback signal $x_f$ is a current that can be *mixed* in *shunt* with the source current.
    - “Shunt” refers to the connection at the input and “series” refers to the connection at the output.
  - Characteristics:
    - Stabilize the current gain
    - Lower input resistance
      (shunt connection at the input)
    - Higher output resistance
      (series connection at the output)

---

Current-sampling shunt-mixing (shunt-series) topology

Example

$I_s \uparrow \Rightarrow i_1 (I_b_1) \uparrow \Rightarrow I_c \uparrow \Rightarrow V_c \downarrow \Rightarrow I_c_2 (I_b) \downarrow \Rightarrow I_c_2 (I_b / \alpha) \downarrow \Rightarrow I_f \uparrow \Rightarrow i_2 (I_b_2) \downarrow \Rightarrow \text{negative feedback}
Transconductance Amplifiers

- Characteristics:
  - Input signal: voltage  output signal: current
  - Voltage-controlled current source
  - High input impedance
  - High output impedance

- Feedback topology
  - Current-sampling series-mixing (series-series) topology
    - The feedback network samples the output current, and the feedback signal $x_f$ is a voltage that can be mixed with the source voltage in series.
    - “Series” refers to the connection at the input and “series” refers to the connection at the output.
  - Characteristics:
    - Stabilize the transconductance gain
    - Higher input resistance
      (series connection at the input)
    - Higher output resistance
      (series connection at the output)

- Current-sampling series-mixing (series-series) topology

- Example
**Transresistance Amplifiers**

- **Characteristics:**
  - Input signal: current     output signal: voltage
  - Current-controlled voltage source
  - Low input impedance
  - Low output impedance

- **Feedback topology**
  - Voltage-sampling shunt-mixing (shunt-shunt) topology
    - The feedback network samples the output voltage, and the feedback signal $x_f$ is a current that can be mixed in with shunt the source current.
    - “Shunt” refers to the connection at the input and “shunt” refers to the connection at the output.
  - Characteristics:
    - Stabilize the transresistance gain
    - Lower input resistance
      (shunt connection at the input)
    - Lower output resistance
      (shunt connection at the output)

**Example**

\[ I_s \uparrow \rightarrow i_i \uparrow \rightarrow V_o \downarrow \rightarrow I_f \uparrow \rightarrow i_i \downarrow \rightarrow \text{negative feedback} \]
Two-Port Network Parameters

- **y parameters**

  ![Equivalent circuit for y parameters](image)

  $$y_{11} = \frac{I_1}{V_1} |_{I_2 = 0}$$
  $$y_{12} = \frac{I_1}{V_2} |_{I_2 = 0}$$
  $$y_{21} = \frac{I_2}{V_1} |_{I_1 = 0}$$
  $$y_{22} = \frac{I_2}{V_2} |_{I_1 = 0}$$

- **z parameters**

  ![Equivalent circuit for z parameters](image)

  $$z_{11} = \frac{V_1}{I_1} |_{I_2 = 0}$$
  $$z_{12} = \frac{V_1}{I_2} |_{I_1 = 0}$$
  $$z_{21} = \frac{V_2}{I_1} |_{I_2 = 0}$$
  $$z_{22} = \frac{V_2}{I_2} |_{I_1 = 0}$$
**h parameters**

Equivalent circuit:

\[ V_1 = h_{11} I_1 + h_{12} I_2 \]
\[ I_2 = h_{21} I_1 + h_{22} V_2 \]

\[ h_{11} = \frac{V_1}{I_1} \bigg|_{V_2 = 0} \]
\[ h_{12} = \frac{V_1}{I_2} \bigg|_{V_2 = 0} \]
\[ h_{21} = \frac{I_1}{V_2} \bigg|_{I_2 = 0} \]
\[ h_{22} = \frac{I_2}{V_2} \bigg|_{I_2 = 0} \]

**g parameters**

Equivalent circuit:

\[ I_1 = g_{11} V_1 + g_{12} I_2 \]
\[ V_2 = g_{21} V_1 + g_{22} I_2 \]

\[ g_{11} = \frac{I_1}{V_1} \bigg|_{I_2 = 0} \]
\[ g_{12} = \frac{I_1}{I_2} \bigg|_{V_2 = 0} \]
\[ g_{21} = \frac{V_2}{V_1} \bigg|_{I_2 = 0} \]
\[ g_{22} = \frac{V_2}{I_2} \bigg|_{V_2 = 0} \]
The Series-Shunt Feedback Amplifier

The Ideal Situation of the Series-Shunt Feedback Amplifier

- Ideal structure

- $A$ circuit: a unilateral open-loop amplifier
  - $R_i$: input resistance
  - $A$: voltage gain
  - $R_o$: output resistance

- $\beta$ circuit: an ideal voltage-sampling series-mixing feedback network
  - The source and load resistances are included inside the $A$ circuit.
  - The $\beta$ circuit does not load the $A$ circuit. $\Rightarrow$ Do not change $A \approx V_o / V_i$. 
1. **Equivalent circuit**

2. **The close-loop gain** \( A_f \equiv \frac{V_o}{V_s} = \frac{A}{1 + A\beta} \) (\( A \) and \( \beta \) have reciprocal units)

3. **Input resistance**

\[
R_{if} = \frac{V_s}{I_i} = \frac{V_s}{V_i / R_i} = R_i \frac{V_s}{V_i} = R_i \frac{V_i + \beta V_o}{V_i} = R_i \frac{V_i + \beta AV_o}{V_i} = R_i (1 + A\beta)
\]

\( \Rightarrow \) The series-mixing increases the input resistance by \((1 + A\beta)\).

General form \( Z_{if} (s) = Z_{if} (s)[1 + A(s)/\beta(s)] \)

4. **Output resistance** \((V_s = 0)\)

Output resistance \( R_{of} \equiv \frac{V_o}{I} \)

where \( I = \frac{V_t - AV_o}{R_o} \) and \( V_i = -V_f = -\beta V_o = -\beta V_t \) (for \( V_s = 0 \))

\( \Rightarrow \) \( I = \frac{V_t + A\beta V_i}{R_o} \)

\( \Rightarrow \) \( R_{of} = \frac{V_o}{I} = \frac{R_o}{1 + A\beta} \)

The voltage-sampling feedback reduces the output resistance by \((1 + A\beta)\).

General form \( Z_{of} (s) = \frac{Z_o (s)}{1 + A(s)/\beta(s)} \)
The Practical Situation

Derivation of the $A$ circuit and $\beta$ circuit for the series-shunt feedback amplifier.

1. Block diagram of a practical series-shunt feedback amplifier

![Block Diagram](image)

$R_{if}$ and $R_{of}$ are the input and output resistances, respectively, of the feedback amplifier, including $R_s$ and $R_L$.

The actual input and output resistances of the feedback amplifier usually exclude $R_s$ and $R_L$:

$$R_{in} = R_{if} - R_s \quad \text{and} \quad R_{out} = \frac{1}{\frac{1}{R_{of}} - \frac{1}{R_L}}$$

2. The circuit in 1 with the feedback network represented by its $h$ parameters.

![h-Parameter Circuit](image)

- The source and load resistances should be lumped with the basic amplifier.
- In addition, the resistances in the feedback should be also lumped with the basic amplifier.
- $h$-parameter two-port feedback network:
  - Neglect $h_{21}$, the forward transmission effect of the feedback network, and thus omit the controlled source $h_{21}I_1$. 

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The circuit in (a) after neglecting \( h_{21} \).

Determine \( \beta \):

\[
\beta = h_{12} \equiv \frac{V_1}{V_{21}} | I_1 = 0
\]

(port 1 open-circuited)

Summary of the Series-Shunt Feedback Amplifier

(a) The \( A \) circuit is

(a) The \( A \) circuit is

where \( R_{11} \) is obtained from

and \( R_{22} \) is obtained from

and the gain \( A \) is defined

\[
A = \frac{V_o}{V_i}
\]

(b) \( \beta \) is obtained from

\[
\beta = \frac{V_0}{V_1} | I_e = 0
\]
**Example 8.1** An op amp connected in the noninverting configuration

The op amp has an open-loop gain $\mu$, a differential input resistance $R_{id}$, and an output resistance $r_o$. Find expressions for $A$, $\beta$, the close-loop gain $V_o / V_s$, the input resistance $R_{in}$, and the output resistance $R_{out}$ ($\mu = 10^4$, $R_{id} = 100k\Omega$, $r_o = 1k\Omega$, $R_L = 2k\Omega$, $R_1 = 1k\Omega$, $R_2 = 1M\Omega$, and $R_s = 10k\Omega$).

The feedback network consists $R_2$ and $R_1$. This network samples the output voltage $V_o$ and provides a voltage signal (across $R_1$) that mixed in series with the input source $V_s$.

- **$A$ circuit:**

$$A \approx \frac{V'_o}{V'_i} = \frac{\mu}{[R_L || (R_1 + R_2)]} = \frac{R_{id}}{R_{id} + R_s + (R_1 || R_2)} \Rightarrow A \approx 6000 \text{ V/V}$$

- **$\beta$ circuit:**

$$\beta \equiv \frac{V'_f}{V'_o} = \frac{R_1}{R_1 + R_2} \approx 10^{-3} \text{ V/V}$$
Voltage gain: \[ A_f \equiv \frac{V_o}{V_s} = \frac{A}{1 + A\beta} = \frac{6000}{7} = 857 \ V/V \]

Input resistance: \[ R_{if} = R_i(1 + A\beta) \] where \( R_i \) is the input resistance of the \( A \) circuit.
\[ R_i = R_s + R_{id} + (R_1 \parallel R_2) \approx 111\, \Omega \]
\[ R_{if} = 111 \times 7 = 777\, \Omega \]
\[ R_{in} = R_{if} - R_s = 739\, \Omega \]

Output resistance: \[ R_{of} = \frac{R_o}{1 + A\beta} \] where \( R_o \) is the output resistance of the \( A \) circuit.
\[ R_o = r_o \parallel R_L \parallel (R_1 + R_2) \approx 667 \ \Omega \]
\[ R_{of} = \frac{667}{7} = 95.3 \ \Omega \]
\[ \therefore R_{of} = R_{out} \parallel R_L \]
\[ \therefore R_{out} \approx 100 \ \Omega \]
The Ideal Situation of the Series-Series Feedback Amplifier

- Ideal structure

- A circuit: a unilateral open-loop amplifier
  - $R_i$: input resistance
  - $A$: transconductance gain
  - $R_o$: output resistance

- $\beta$ circuit: an ideal current-sampling series-mixing feedback network
  - The source and load resistances are included inside the A circuit.
  - The $\beta$ circuit does not load the A circuit. $\Rightarrow$ Do not change $A \approx I_o / V_i$

- Equivalent circuit

1. The close-loop gain $A_f \equiv \frac{I_o}{V_s} = \frac{A}{1 + A\beta}$ ($A$ and $\beta$ have reciprocal units)

2. Input resistance

$$R_f = \frac{V_s}{I_i} = \frac{V_s}{V_i / R_i} = R_i \frac{V_s}{V_i} = \frac{V_i V_o + \beta V_i}{V_i} = \frac{V_i + \beta AV_i}{V_i} = R_i(1 + A\beta)$$

$\Rightarrow$ The series-mixing increases the input resistance by $(1 + A\beta)$. General form $Z_{if}(s) = Z_i(s)[1 + A(s)/\beta(s)]$
Output resistance \((V_s = 0)\)

![Circuit Diagram]

Output resistance \(R_{of} \equiv \frac{V}{I_t}\)

In this case, \(V_i = -V_f = -\beta I_o = -\beta I_t\) (for \(V_s = 0\))

\[
\Rightarrow \quad V = (I_t - AV_i)R_o = (I_t + A\beta I_t)R_o
\]

\[
\Rightarrow \quad R_{of} = \frac{V}{I_t} = R_o(1 + A\beta)
\]

The voltage-sampling feedback increases the output resistance by \((1 + A\beta)\).

General form \(Z_{of}(s) = Z_o(s)[1 + A(s)\beta(s)]\)

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**The Practical Situation**

Derivation of the \(A\) circuit and \(\beta\) circuit for the series-series feedback amplifier.

- **Block diagram of a practical series-shunt feedback amplifier**

![Block Diagram]

\(R_{if}\) and \(R_{of}\) are the input and output resistances, respectively, of the feedback amplifier, including \(R_s\) and \(R_L\).

The actual input and output resistances of the feedback amplifier usually exclude \(R_s\) and \(R_L\): \[R_{in} = R_{if} - R_s\] \[R_{out} = R_{of} - R_L\]
The circuit in ① with the feedback network represented by its $z$ parameters.

The source and load resistances should be lumped with the basic amplifier.

In addition, the resistances in the feedback should be also lumped with the basic amplifier.

$z$-parameter two-port feedback network:
Neglect $z_{21}$, the forward transmission effect of the feedback network, and thus omit the controlled source $z_{21}I_1$.

The circuit in ② after neglecting $z_{21}$.

Determine $\beta$:  \[ \beta = z_{12} \equiv \left| \frac{V_1}{I_2} \right| \bigg|_{I_1 = 0} \]  (port 1 open-circuited)
Example 8.2 A feedback triple is composed of three gain stages with series-series feedback

Assume that the bias circuit causes $I_{C1} = 0.6\, mA$, $I_{C2} = 1\, mA$, and $I_{C3} = 4\, mA$. Using these values and assuming $h_{fe} = 100$ and $r_o = \infty$, find the open-loop gain $A$, the feedback factor $\beta$, the closed loop gain $A_f = I_o / V_s$, the voltage gain $V_o / V_s$, the input resistance $R_{in} = R_{if}$, and the output resistance $R_{of}$ (between nodes $Y$ and $Y'$). Now, if $r_o$ of $Q_3$ is $25\, k\Omega$, estimate an approximate value of the output resistance $R_{out}$. 

[Diagram of a feedback amplifier with labeled components]
A circuit:

\[ V_{el} = \frac{-\alpha (R_{c1} || R_{e2})}{r_e + [R_{E1} (R_p + R_{E2})]} = -14.92 \text{ V/V} \]
\[ V_{e2} = -g_{m2} \left( [R_{C2} (h_{fe} + 1)] [r_e3 + (R_{E2} (R_p + R_{E1})]] \right) = -131.2 \text{ V/V} \]
\[ I_o = \frac{I_{e3}}{r_e3 + (R_{E2} (R_p + R_{E1}))) = 10.6 \text{ mA/V} \]

\[ A \approx \frac{I_o}{V_i} = \frac{V_{el}}{V_i} \times \frac{V_{e2}}{V_{e1}} \times \frac{I_o}{V_i} = 20.7 \text{ A/V} \]

\[ \beta = \frac{V_f'}{I_o} = \frac{R_{E2}}{R_{E2} + R_F + R_{E1}} = 11.9 \Omega \]

- Closed-loop gain: \[ A_f = \frac{I_o}{V_s} = \frac{A}{1 + A\beta} = \frac{20.7}{1 + 20.7 \times 11.9} = 83.7 \text{ mA/V} \]
- Voltage gain: \[ \frac{V_o}{V_s} = -\frac{I_c R_{C3}}{V_s} \approx -A_f R_{C3} = -83.7 \times 10^{-3} \times 600 = -50.2 \text{ V/V} \]
- Input resistance of the A circuit: \[ R_i = (h_{fe} + 1) [r_e1 + (R_{E1} (R_p + R_{E2}))] = 13.65 \text{ k\Omega} \]
- Input resistance of the feedback amplifier: \[ R_{if} = R_i (1 + A\beta) = 3.34 \text{ M\Omega} \]
- Output resistance \( R_o \) of the A circuit: The resistance looking between \( Y \) and \( Y' \):
\[ R_o = [R_{E2} (R_p + R_{E1})] + r_e3 + \frac{R_{C2}}{h_{fe} + 1} = 143.9 \Omega \]
- Output resistance of the feedback amplifier:
\[ R_{of} = R_o (1 + A\beta) = 35.6 \text{ k\Omega} \]
Find the output resistance from the equivalent circuit.

\[ R_{out} = r_{o3} + (1 + g_{m3}r_{o3})(R_{of \parallel r_{\pi3}}) \]
\[ \approx r_{o3}(1 + g_{m3}(R_{of \parallel r_{\pi3}})) \]
\[ = 25(1 + 100(35.6 \parallel 1)) \]
\[ = 2.5 \, \text{M\Omega} \]

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**Homework #1**

- Problems 6, 14, 24, 25, 27, 37
The Shunt-Shunt Feedback Amplifier

The Ideal Situation of the Shunt-Shunt Feedback Amplifier

- Ideal structure

- $A$ circuit: a unilateral open-loop amplifier
  - $R_i$: input resistance  
  - $A$: transresistance gain  
  - $R_o$: output resistance

- $\beta$ circuit (transconductance): an ideal voltage-sampling shunt-mixing feedback network

  The source and load resistances are included inside the $A$ circuit.

  The $\beta$ circuit does not load the $A$ circuit.  $\not\Rightarrow$ Do not change $A \approx V_o / I_i$. 

1. Equivalent circuit

2. The close-loop gain \( A_f \equiv \frac{V_o}{I_s} = \frac{A}{1 + A\beta} \) (A and \( \beta \) have reciprocal units)

3. Input resistance

\[
R_{if} = \frac{V_i}{I_s} = I_i R_i = R_i \frac{I_i}{I_s} = R_i \frac{I_i}{I_i + I_f} = R_i \frac{I_i}{I_i + \beta V_o} = R_i \frac{I_i}{I_i + \beta A I_i} = \frac{R_i}{1 + A\beta}
\]

\( \Rightarrow \) The shunt-mixing reduces the input resistance by \( (1 + A\beta) \).

General form \( Z_{if}(s) = \frac{Z_i(s)}{1 + A(s)\beta(s)} \)

4. Output resistance \( (V_s = 0) \)

Output resistance \( R_{of} \equiv \frac{V_o}{I} \)

where \( I = \frac{V_t - A I_i}{R_o} \) and \( I_i = -I_f = -\beta V_o = -\beta V_t \) (for \( I_s = 0 \))

\( \Rightarrow \)

\[
I = \frac{V_t + A\beta V_t}{R_o}
\]

\( \Rightarrow \)

\[
R_{of} = \frac{V_o}{I} = \frac{R_o}{1 + A\beta}
\]

The voltage-sampling feedback reduces the output resistance by \( (1 + A\beta) \).

General form \( Z_{of}(s) = \frac{Z_o(s)}{1 + A(s)\beta(s)} \)
**The Practical Situation**

Derivation of the $A$ circuit and $\beta$ circuit for the series-shunt feedback amplifier.

1. Block diagram of a practical shunt-shunt feedback amplifier

2. The circuit in 1 with the feedback network represented by its $y$ parameters.

$R_{if}$ and $R_{of}$ are the input and output resistances, respectively, of the feedback amplifier, including $R_s$ and $R_L$.

The actual input and output resistances of the feedback amplifier usually exclude $R_s$ and $R_L$:

$$R_{in} = \frac{1}{\frac{1}{R_{if}} - \frac{1}{R_s}} \quad \text{and} \quad R_{out} = \frac{1}{\frac{1}{R_{of}} - \frac{1}{R_L}}$$

- The source and load resistances should be lumped with the basic amplifier.
- In addition, the resistances in the feedback should be also lumped with the basic amplifier.
- $y$-parameter two-port feedback network:
  - Neglect $y_{21}$, the forward transmission effect of the feedback network, and thus omit the controlled source $y_{21}I_1$. 
The circuit in 2 after neglecting $y_{21}$.

Determine $\beta$: $\beta = y_{12} \equiv \left. I_2 \right|_{V_i = 0}$ (port 1 short-circuited)

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**Summary of the Shunt-Shunt Feedback Amplifier**

(a) The $A$ circuit is

(b) $\beta$ is obtained from

$\beta = \left. \frac{V_o}{V_i} \right|_{V_i = 0}$
Example 8.3  A feedback circuit of the shunt-shunt type

Determine the small-signal voltage gain \( V_o / V_s \), the input resistance \( R_{in} \), and the output resistance \( R_{out} = R_{of} \). The transistor has \( \beta = 100 \).

- dc analysis:

\[
\begin{align*}
V_C &= 0.7 + (I_B + 0.07)47 = 3.99 + 47I_B \\
12 - V_C &= (\beta + 1)I_B + 0.07 \\
I_B &\approx 0.015 \text{mA}, I_C = 1.5 \text{mA}, \text{ and } V_C = 4.5 \text{V}
\end{align*}
\]

- Small-signal analysis

A circuit:

\( I_i = \frac{V_s}{R_s} \)

\( I_i = \frac{V_s}{R_s} \parallel \frac{R_f}{r_\pi} = 1.4 \text{ k\Omega} \)

\( R_o = R_C \parallel R_f = 4.27 \text{ k\Omega} \)

\( V_x = I_i \left( R_s \parallel R_f \parallel r_\pi \right) \)

\( V'_o = -g_m V_x \left( R_f \parallel R_C \right) \)

\( A = \frac{V'_o}{I_i} = \frac{-g_m (R_f \parallel R_C) (R_s \parallel R_f \parallel r_\pi)}{R_x} \approx -358.7 \text{ k\Omega} \)
- Determine $\beta$:

\[ \beta \equiv \frac{I_f}{V_o} = -\frac{1}{R_f} = -\frac{1}{47\text{k}\Omega} \]

- Determine the closed-loop characteristics:

Closed-loop gain
\[ A_f \equiv \frac{V_o}{I_s} = \frac{A}{1 + A\beta} = \frac{-358.7}{1 + 358.7/47} = -41.6 \text{k}\Omega \]

Since $V_s = I_s R_s$, thus the voltage gain
\[ \frac{V_o}{V_s} = \frac{V_o}{I_s R_s} = \frac{-41.6}{10} \approx -4.16 \text{ V/V} \]

Input resistance with feedback
\[ R_i = \frac{R_i}{1 + A\beta} = \frac{1.4}{8.63} = 162.2 \Omega \]

Output resistance with feedback
\[ R_o = \frac{R_o}{1 + A\beta} = \frac{4.27}{8.63} = 495 \Omega \]

---

The Shunt-Series Feedback Amplifier
The Ideal Situation of the Shunt-Series Feedback Amplifier

- Ideal structure

- A circuit: a unilateral open-loop amplifier
  - $R_i$: input resistance
  - $A$: current gain
  - $R_o$: output resistance

- $\beta$ circuit: an ideal current-sampling shunt-mixing feedback network
  - The source and load resistances are included inside the $A$ circuit.
  - The $\beta$ circuit does not load the $A$ circuit. $\Rightarrow$ Do not change $A \approx I_o / I_i$.

- Equivalent circuit

1. The close-loop gain $A_f = \frac{I_o}{I_s} = \frac{A}{1 + A\beta}$ ($A$ and $\beta$ have reciprocal units)

2. Input resistance

   $$R_f = \frac{V_i}{I_s} = \frac{I_f}{I_s} = \frac{R_i}{I_i + I_f} = \frac{R_i}{I_i + \beta I_i} = \frac{R_i}{1 + A\beta}$$

   $\Rightarrow$ The series-mixing reduces the input resistance by $(1 + A\beta)$.

General form $Z_f(s) = \frac{Z_i(s)}{1 + A(s)\beta(s)}$
Output resistance \((V_s = 0)\)

In this case, \(I_i = -I_f = -\beta I_o = -\beta I_t\) (for \(I_s = 0\))

\[
V = (I_t - AI_t)R_o = (I_t + A\beta I_t)R_o
\]

\[
R_{of} = \frac{V}{I_t} = R_o(1 + A\beta)
\]

The current-sampling feedback increases the output resistance by \((1 + A\beta)\).

General form \(Z_{of}(s) = Z_o(s)[1 + A(s)/\beta(s)]\)

---

**The Practical Situation**

Derivation of the \(A\) circuit and \(\beta\) circuit for the series-shunt feedback amplifier.

Block diagram of a practical shunt-series feedback amplifier

\(R_{if}\) and \(R_{of}\) are the input and output resistances, respectively, of the feedback amplifier, including \(R_s\) and \(R_L\).

The actual input and output resistances of the feedback amplifier usually exclude \(R_s\) and \(R_L\):

\[
R_{in} = \frac{1}{\frac{1}{R_{if}} - \frac{1}{R_s}} \quad \text{and} \quad R_{out} = R_{of} - R_L
\]
The circuit in 1 with the feedback network represented by its $g$ parameters.

![Circuit Diagram](image)

- The source and load resistances should be lumped with the basic amplifier.
- In addition, the resistances in the feedback should be also lumped with the basic amplifier.
- $g$-parameter two-port feedback network:
  - Neglect $g_{21}$, the forward transmission effect of the feedback network, and thus omit the controlled source $g_{21} V_1$.

Determine $\beta$:

$$\beta = g_{12} \equiv \left. \frac{I_1}{I_2} \right|_{V_1 = 0}$$  \text{(port 1 short-circuited)}

The circuit in 2 after neglecting $g_{21}$.

![Circuit Diagram](image)
Example 8.4  A feedback circuit of the shunt-series type

Find the current gain $I_{out}/I_{in}$, the input resistance $R_{in}$, and the output resistance $R_{out}$. The transistor has $\beta = 100$ and $V_A = 75V$.

- dc analysis: (neglect the effect of finite transistor $\beta$ and $V_A$)

$$V_{E1} \approx 12 - \frac{15}{100 + 15} = 1.57 \text{ V}$$
$$V_{E2} \approx 2 - 0.7 = 1.3 \text{ V}$$
$$V_{C1} \approx 12 - 10 \times 1 = 2 \text{ V}$$
$$V_{C2} \approx 12 - 0.4 \times 8 = 8.8 \text{ V}$$
$$I_{E1} \approx \frac{0.87}{0.87} = 1 \text{ mA}$$
$$I_{E2} \approx \frac{1.3}{3.4} = 0.4 \text{ mA}$$
**Small-signal equivalent circuit**

\[ V_{x1} = I'_i \left[ R_e \parallel \left( R_{E2} + R_f \right) \parallel R_f \right] \]
\[ V_{b2} = -g_m V_{x1} \left\{ I_{o1} \left[ R_{C1} \parallel R_{x2} \right] + \left( \beta + 1 \right) \left( R_{E2} \parallel R_f \right) \right\} \]
\[ I'_o \approx \frac{V_{b2}}{r_{e2} + \left( R_{E2} \parallel R_f \right)} \Rightarrow A = \frac{I'_o}{I'_i} \approx -201.45 \text{ A/A} \]

- **Determine** \( \beta \):
  \[ \beta = \frac{I'_f}{I'_o} = -\frac{R_{E2}}{R_{E2} + R_f} = -0.254 \]

- **Determine the closed-loop characteristics**:

Find input/output impedance:

\[ R_i = R_e \parallel \left( R_{E2} + R_f \right) \parallel R_f \parallel r_{x1} = 1.535 \text{ k}\Omega \]
\[ R_o = (R_{E2} \parallel R_f) + r_{e2} + \frac{R_{C1} \parallel r_{s1}}{\beta + 1} = 2.69 \text{ k}\Omega \]
\[ \Rightarrow R_{if} = \frac{R_i}{1 + A\beta} = 29.5 \text{ \Omega} \]
\[ \Rightarrow R_{of} = R_o (1 + A\beta) \approx 140.1 \text{ k}\Omega \]
\[ R_{in} = \frac{1}{1/R_{if} - 1/R_s} \approx 29.5 \text{ \Omega} \]
\[ \Rightarrow R_{out} = r_{o2} \left[ 1 + g_m \left( r_{x2} \parallel R_{of} \right) \right] \approx 18.1 \text{ M}\Omega \]

Find the required current gain:

\[ \frac{I_{out}}{I_{in}} = \frac{I_{out}}{I_s} \approx \frac{R_{C2}}{R_L + R_{C2}} \frac{I_s}{I_s} = \frac{R_{C2}}{R_L + R_{C2}} \frac{I_s}{I_s} = -3.44 \text{ A/A} \]
### Summary of Relationships for the Four Feedback Amplifier Topologies

<table>
<thead>
<tr>
<th>Feedback Amplifier</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$y_1$</th>
<th>$y_2$</th>
<th>$A$</th>
<th>$B$</th>
<th>Source Form</th>
<th>Loading of Feedback Network is Obtained</th>
<th>To Find $\beta_1$, Apply to Port 2 of Feedback Network</th>
<th>$Z_{d}$</th>
<th>$Z_{df}$</th>
<th>Refer to Figs.</th>
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<tbody>
<tr>
<td>Series–shunt (voltage amplifier)</td>
<td>$V_i$</td>
<td>$V_C$</td>
<td>$V_y$</td>
<td>$V_y$</td>
<td>$V_y$</td>
<td>$V_y$</td>
<td>Thévenin</td>
<td>By short-circuiting port 2 of feedback network</td>
<td>A voltage and find the open-circuit voltage at port 1</td>
<td>$Z/(1 + AB)$</td>
<td>$Z/a$</td>
<td>8.4(a)</td>
</tr>
<tr>
<td>Shunt–series (current amplifier)</td>
<td>$I_i$</td>
<td>$I_L$</td>
<td>$I_y$</td>
<td>$I_y$</td>
<td>$I_y$</td>
<td>$I_y$</td>
<td>Norton</td>
<td>By open-circuiting port 1 of feedback network</td>
<td>$1 + AB$</td>
<td>$Z/(1 + AB)$</td>
<td>$Z/(1 + AB)$</td>
<td>8.8</td>
</tr>
<tr>
<td>Series–series (transconductance amplifier)</td>
<td>$V_i$</td>
<td>$V_L$</td>
<td>$V_y$</td>
<td>$V_y$</td>
<td>$V_y$</td>
<td>$V_y$</td>
<td>Thévenin</td>
<td>By open-circuiting port 2 of feedback network</td>
<td>$Z/(1 + AB)$</td>
<td>$1 + AB$</td>
<td>$Z/(1 + AB)$</td>
<td>8.22</td>
</tr>
<tr>
<td>Shunt–shunt (transresistance amplifier)</td>
<td>$I_i$</td>
<td>$I_L$</td>
<td>$I_y$</td>
<td>$I_y$</td>
<td>$I_y$</td>
<td>$I_y$</td>
<td>Norton</td>
<td>By short-circuiting port 1 of feedback network</td>
<td>$Z/(1 + AB)$</td>
<td>$1 + AB$</td>
<td>$Z/(1 + AB)$</td>
<td>8.4(b)</td>
</tr>
</tbody>
</table>

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**Determining The Loop Gain**

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Microelectrics (III) 8-71 Ching-Yuan Yang / EE, NCHU
Determining the Loop Gain

Loop gain $A\beta$ is a very important quantity that characterizes a feedback loop.

1. Find the loop gain:

Let the external source $x_s = 0$.

2. Open the feedback loop by breaking the connection of $x_o$ to the feedback network and apply a test signal $x_t$.

3. Determine the loop gain:

$$x_f = \beta x_t \Rightarrow x_i = -x_f = -\beta x_t \Rightarrow x_o = A x_i = -A\beta x_t \Rightarrow \text{Loop gain } A\beta = -x_o / x_t$$

The loop gain $A\beta$ is given by the negative of the ratio of the returned signal to the applied test signal.

A conceptual feedback loop is broken at XX’ and a test voltage $x_t$ is applied. The impedance $Z_t$ is equal to that previously seen looking to the left of XX’. The loop gain $A\beta = -V_r / V_t$, where $V_r$ is the returned voltage.

As an alternative, $A\beta$ can be determined by finding the open-circuit transfer function $T_{oc}$ and the short-circuit transfer function $T_{sc}$ and combining them as indicated.

$A\beta = -\frac{1}{T_{oc}} + \frac{1}{T_{sc}}$
Closed-loop circuit:

Break XX’ to determine the loop gain $A\beta$:

\[ V_x + - A_x V_x R_x R_t \]
\[ V_c + - V_t \]
\[ V_x + - A_x V_x R_x R_t \]

\[ V_r = A_x V_t \frac{R_t}{R_x + R_t} \]
\[ A\beta = \frac{V_r}{V_t} = -A_x \frac{R_t}{R_x + R_t} \]

Break YY’ to determine $T_{oc}$ and $T_{sc}$:

Open-circuit transfer function $T_{oc}$

\[ V_x + - A_x V_x R_x R_t \]
\[ V_c + - V_t \]
\[ V_x + - A_x V_x R_x R_t \]

\[ V_{oc} = A_x V_t \]
\[ T_{oc} = \frac{V_{oc}}{V_t} = A_x \]

Short-circuit transfer function $T_{sc}$

\[ V_x + - A_x V_x R_x R_t \]
\[ I_c + - V_t \]
\[ I_c + - A_x V_x R_x R_t \]

\[ V_x = I_t R_t \]
\[ I_{sc} = \frac{A_x V_x}{R_x} = \frac{A_x I_t R_t}{R_x} \]
\[ T_{sc} = \frac{I_{sc}}{I_t} = A_x \frac{R_t}{R_x} \]

Thus,

\[ \frac{1}{T_{oc}} + \frac{1}{T_{sc}} = -\frac{1}{A_x} + \frac{1}{A_x R_t} = A\beta \]
Equivalence of Circuits from a Feedback-Loop Point of View

Break XX' to determine the loop gain $A\beta$:

$$V_r = -\mu V_1 \left[ \frac{R_L}{R_2 + R_1} \right] \left[ \frac{R_1}{R_{id} + R} \right] + r_o \left[ \frac{R_1}{R_{id} + R} \right] + \frac{R_{id}}{R + R}$$

$$L = A\beta = -\frac{V_r}{V_1} = -\frac{V_r}{V_1} = \mu \left[ \frac{R_L}{R_2 + R_1} \right] \left[ \frac{R_1}{R_{id} + R} \right] + r_o \left[ \frac{R_1}{R_{id} + R} \right] + \frac{R_{id}}{R + R}$$

The Stability Problem
Stability Problem of a Feedback Amplifier

The closed-loop transfer function of a feedback amplifier

\[ A_f(s) = \frac{A(s)}{1 + A(s)\beta(s)} \]

where \( A(s) \): the open-loop transfer function
\( \beta(s) \): the feedback transfer function

For physical frequencies \( s = j\omega \),

\[ A_f(j\omega) = \frac{A(j\omega)}{1 + A(j\omega)\beta(j\omega)} \quad \Leftrightarrow \quad L(j\omega) = A(j\omega)\beta(j\omega) = |A(j\omega)\beta(j\omega)|e^{j\phi(\omega)} \]

- If for \( s = j\omega_0 \), \( L(j\omega_0) = -1 \), then the closed-loop gain approaches infinity at \( \omega_0 \).
  Under this condition, the circuit amplifies its own noise components at \( \omega_0 \) indefinitely.

- Barkhausen criteria: If a negative-feedback circuit has a loop gain that satisfies two conditions:
  \[ |L(j\omega_0)| \geq 1 \quad \text{and} \quad \angle L(j\omega_0) = 180^\circ, \]
  then the circuit may oscillate at \( \omega_0 \).

In order to ensure oscillation in the presence of temperature and process variations, we typically choose the loop gain to be at least twice or three times the required value.

- Oscillations could occur in a negative-feedback amplifier, we wish to find methods to prevent their occurrence.

Nyquist Plot

Nyquist criterion:
- If the intersection occurs to the left of \((-1, 0)\), the magnitude of loop gain at this frequency is greater than unity and the amplifier will be unstable.
- If the intersection occurs to the right of \((-1, 0)\), the amplifier will be stable.
Effect Of Feedback On The Amplifier Poles

Stability and Pole Location

Consider an amplifier with a pole pair at \( s = \sigma_0 \pm j\omega_n \).

\[
\Rightarrow \quad v(t) = e^{\sigma_0 t} \left[ e^{j\omega_n t} + e^{-j\omega_n t} \right] = 2 e^{\sigma_0 t} \cos (\omega_n t)
\]

Relationship between pole location and transient response:

The existence of any right-half-plane poles results in instability.
Poles of the Feedback Amplifier

The closed-loop transfer function of a feedback amplifier

\[ A_f(s) = \frac{A(s)}{1 + A(s)\beta(s)} \]

where \( A(s) \): the open-loop transfer function
\( \beta(s) \): the feedback transfer function

Find poles: The poles of the feedback amplifier are the zeros of \( 1 + A(s)\beta(s) \).

\[ 1 + A(s)\beta(s) = 0 \] (the characteristic equation of the feedback loop.)

It should therefore be apparent that applying feedback to an amplifier changes its poles.

Amplifier with Single-Pole Response

An amplifier whose open-loop transfer function is characterized by a single pole:

\[ A(s) = \frac{A_0}{1 + s/\omega_p} \]

The closed-loop transfer function

\[ A_f(s) = \frac{A_0 / (1 + A_0\beta)}{1 + s/\omega_p(1 + A_0\beta)} \]

<table>
<thead>
<tr>
<th></th>
<th>open-loop</th>
<th>closed-loop</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Midband Gain</strong></td>
<td>( A_0 )</td>
<td>( A_{of} = \frac{A_0}{1 + A_0\beta} )</td>
</tr>
<tr>
<td><strong>Pole</strong></td>
<td>( \omega_p )</td>
<td>( \omega_{pf} = \omega_p(1 + A_0\beta) )</td>
</tr>
</tbody>
</table>

Effect of feedback on the pole location and the frequency response:
For frequencies \( \omega >> \omega_p(1 + A_0\beta) \), \( A_f(s) \approx \frac{A_0/\omega_P}{s} \approx A(s) \)

At such high frequencies the loop gain is much smaller than unity and the feedback is ineffective.

Effect of feedback on the frequency response:

- Applying negative feedback to an amplifier results in extending its bandwidth at the expense of a reduction in gain.
- Since the pole of the closed-loop amplifier never enters the right half of the \( s \) plane, the single-pole amplifier is stable for any value of \( \beta \).
  \( \Rightarrow \) Unconditionally stable

Amplifier with Two-Pole Response

An amplifier whose open-loop transfer function is characterized by two poles:

\[
A(s) = \frac{A_0}{(1 + s/\omega_{p1})(1 + s/\omega_{p2})}
\]

- Close-loop poles

\[
1 + A(s)\beta = 0 \Rightarrow s^2 + s(\omega_{p1} + \omega_{p2}) + (1 + A_0\beta)\omega_{p1}\omega_{p2} = 0 \\
\Rightarrow s = -\frac{\omega_{p1} + \omega_{p2}}{2} \pm \frac{1}{2}\sqrt{(\omega_{p1} + \omega_{p2})^2 - 4(1 + A_0\beta)\omega_{p1}\omega_{p2}}
\]

- Root-locus diagram

\( \Rightarrow \) The feedback amplifier also is unconditionally stable. The maximum phase shift of \( A(s) \) is \( 180^\circ \) (90\(^\circ\) per pole), but this value is reached at \( \omega = \infty \).

\( \Rightarrow \) The open-loop amplifier might have a dominant pole, but this is not necessarily the case for the closed-loop amplifier.

\( \Rightarrow \) As is the case with second-order responses generally, the closed-loop response can show a peak.
The characteristic equation of a second-order network:

\[ s^2 + s \frac{\omega_0}{Q} + \omega_0^2 = 0 \quad \omega_0 : \text{pole frequency} \quad Q : \text{pole Q factor} \]

Definition of \( \omega_0 \) and \( Q \) of a pair of complex conjugate poles:

Determine \( Q \) factor:

\[ Q = \sqrt{\frac{(1 + A_0 \beta) \omega_{p1} \omega_{p2}}{\omega_{p1} + \omega_{p2}}} \]

Normalized magnitude response:

\( Q > 0.7 \), peaking. \( Q \leq 0.7 \), no peaking. \( Q = 0.7 \) (poles at 45\(^\circ\)), maximally flat.

---

**Example 8.5 Positive-feedback circuit**

Find the loop transmission \( L(s) \) and the characteristic equation. Sketch a root-locus diagram for varying \( K \), and find the value of \( K \) that results in a maximally flat response, and the value of \( K \) that makes the circuit oscillate.

Assume that the amplifier has infinite input impedance and output impedance.

**Solution**

The loop transmission:

\[ L(s) = A(s) \beta(s) = \frac{V_r}{V_i} = -KT(s) \]

where \( T(s) \) is the transfer function of the two-port RC network.

\[ T(s) = \frac{V_r}{V_i} = \frac{s(1/CR)}{s^2 + s(3/CR) + (1/CR)^2} \]

Thus

\[ L(s) = \frac{-s(K/CR)}{s^2 + s(3/CR) + (1/CR)^2} \]

The characteristic equation:

\[ 1 + L(s) = 0 \]

\[ s^2 + s \frac{3}{CR} + \left( \frac{1}{CR} \right)^2 - s \frac{K}{CR} = 0 \]

\[ \therefore \text{Pole frequency: } \omega_0 = \frac{1}{CR} \quad \text{Q factor: } Q = \frac{1}{3 - K} \]
Amplifier with Three or More Poles

A value of $A_0 \beta$ exists at which this pair of complex-conjugate poles enters the right half of the s plane, thus causing the amplifier to become unstable.

- An amplifier with three poles:

  Since the amplifier with three poles has a phase shift that reach $-270^\circ$ as $\omega$ reaches $\infty$, there exists a finite frequency $\omega_{180^\circ}$, at which the loop gain has $180^\circ$ phase shift.

- Frequency compensation

  There exists a maximum value for $\beta$ above which the feedback amplifier becomes unstable. Alternatively, there exists a minimum value value for the closed-loop gain $A_{f0}$ below which the amplifier becomes unstable.

  To obtain lower values of the closed-loop gain one needs to alter the loop transfer function $L(s)$. This is the process of frequency compensation.
Stability Study Using Bode Plots

Gain and Phase Margins

Gain margin:
The difference between the value of $|A\beta|$ at $\omega_{180^\circ}$ and unity (in dB).

Phase margin:
The difference between the phase angle at the 0-dB frequency and $180^\circ$.

- The gain margin represents the amount by which the loop gain can be increased while stability is maintained.
- If at the frequency of unity loop-gain magnitude, the phase lag is in excess of $180^\circ$, the amplifier will be unstable.
- Feedback amplifiers are usually designed to have sufficient gain margin and phase margin to allow for the inevitable changes in loop gain with temperature, time, and so on.
Effect of Phase Margin on Closed-Loop Response

\[ A(j\omega) = 1 \times e^{-j\theta} \]

where \( \theta = 180^\circ - \text{phase margin} \)

Close-loop gain (at \( \omega_1 \)):

\[ A_f(j\omega_1) = \frac{A(j\omega_1)}{1 + A(j\omega_1)\beta} = \frac{(1/\beta)e^{-j\theta}}{1 + e^{-j\theta}} \]

\[ \Rightarrow |A_f(j\omega_1)| = \frac{1/\beta}{|1 + e^{-j\theta}|} \]

▷ For a phase margin of \(45^\circ\), \( \theta = 135^\circ \),

\[ |A_f(j\omega_1)| = 1.3 \frac{1}{\beta} \quad \text{Gain peaking} \]

▷ The peaking increase as the phase margin is reduced, eventually reaching \(\infty\) when the phase margin is zero.

▷ The closed-loop gain at low frequencies is approximately \(1/\beta\).

An Alternative Approach for Stability Analysis Using Bode Plot of \(|A|\)

- Assume that \(\beta\) is independent of frequency, we can plot \(20\log(1/\beta)\) as a horizontal straight line on the same plane used for \(20\log|A|\).

\[ 20 \log|A(j\omega)| - 20 \log \frac{1}{\beta} = 20 \log|A\beta| \]

(expressed in dB)

- Study stability by examining the difference between the two plots.

If we wish to evaluate stability for a different feedback factor we simply draw another horizontal straight line at the level \(20\log(1/\beta)\).
Example: Stability Analysis
Using Bode Plot of $|A|$

The open-loop gain of the amplifier

$$20 \log |A(j\omega)| - 20 \log \frac{1}{\beta} = 20 \log |A\beta|$$

We can find

$$A(f) = \frac{10^5}{\left(1 + j \frac{f}{10^5}\right) \left(1 + j \frac{f}{10^6}\right) \left(1 + j \frac{f}{10^7}\right)}$$

and

$$|A| = 100 - 20 \log \sqrt{1 + \left(\frac{f}{10^5}\right)^2}$$

$$-20 \log \sqrt{1 + \left(\frac{f}{10^5}\right)^2} - 20 \log \sqrt{1 + \left(\frac{f}{10^6}\right)^2}$$

$$\phi = \left[\tan^{-1}\left(\frac{f}{10^5}\right) + \tan^{-1}\left(\frac{f}{10^6}\right) + \tan^{-1}\left(\frac{f}{10^7}\right)\right]$$

---

Frequency Compensation
Theory of Frequency Compensation

A': Introduce an additional pole at \( f_D \).
poles: \( f_D, f_{P1}, f_{P2}, f_{P3}, \ldots \)

A'': Move the original low-frequency pole to \( f'_D \).
(i.e., \( f_{P1} \rightarrow f'_D \))
poles: \( f'_D, f_{P2}, f_{P3}, \ldots \)

\[ \beta = 10^{-2} \]
\[ 20 \log \frac{1}{\beta} = 40 \text{ dB} \]

Implementation of Frequency Compensation

- Two-Cascaded gain stages of a multistage amplifier

- Equivalent circuit for the interface between the two stages

\[ f_{P1} = \frac{1}{2\pi C_x R_x} \]

- The circuit with a compensating capacitor \( C_C \) added

\[ f'_D = \frac{1}{2\pi (C_x + C_C) R_x} \]
Miller Compensation and Pole Splitting

- A gain stage in a multistage amplifier before compensation

- A gain stage in a multistage amplifier with a compensating capacitor in the feedback loop

\[
\begin{align*}
V_a &= \frac{(sC_f - g_m)R_1R_2}{1 + s[C_1R_1 + C_2R_2 + C_f(g_mR_1R_2 + R_1 + R_2)] + s^2[C_1C_2 + C_f(C_1 + C_2)]R_1R_2} \\
D(s) &= \left(1 + \frac{s}{\omega'_{p1}}\right)\left(1 + \frac{s}{\omega'_{p2}}\right) = 1 + s\left(\frac{1}{\omega'_{p1}} + \frac{1}{\omega'_{p2}}\right) + \frac{s^2}{\omega'_{p1}\omega'_{p2}} \\
\omega'_{p1} &= \frac{C_1R_1 + C_2R_2 + C_f(g_mR_1R_2 + R_1 + R_2)}{g_mR_2C_fR_1} \approx \frac{1}{g_mR_2C_fR_1} \quad (8.87) \\
\omega'_{p2} &\approx \frac{g_mC_f}{C_1C_2 + C_f(C_1 + C_2)} \quad (8.88)
\end{align*}
\]

Before Compensation

After Compensation

Pole splitting
Example 8.6: Frequency Compensation

The open-loop transfer function of the operational amplifier:

\[ A(f) = \frac{10^6}{1 + j \frac{f}{10^6}} \left( 1 + j \frac{f}{10^6} \right) \]

We wish to compensate the op amp so that the closed-loop amplifier with resistive feedback is stable for any gain (that is, for \( \beta \) up to unity) and \( PM (\text{phase margin}) \geq 45^\circ \).

Assume that the op amp circuit includes a stage such as that of Fig.a with \( C_1 = 100 \text{pF} \), \( C_2 = 5 \text{pF} \), and \( g_m = 40 \text{mA/V} \), that the pole at \( f_{P1} \) is caused by by the input circuit of that stage, and that the pole is connected either 1 between the input node B and ground or 2 in the feedback path of the transistor.

Fig.a:

Find the compensating capacitor \( C_C \) connected across the input terminals of the transistor stage.

(a) Determine \( R_1 \) and \( R_2 \):

\[ f_{P1} = 0.1 \text{MHz} = \frac{1}{2\pi C_1 R_1} \quad R_1 = \frac{10^5}{2\pi} \ \Omega \]

\[ f_{P2} = 1 \text{MHz} = \frac{1}{2\pi C_2 R_2} \quad \Rightarrow \quad R_2 = \frac{10^5}{\pi} \ \Omega \]

(b) Determine \( f'_D \):

1st pole from \( f_{P1} \) to \( f'_D \):

\[ f'_D = \frac{1}{2\pi(C_1 + C_C)R_1} \quad \text{and the 2nd pole remains unchanged.} \]

\( PM \geq 45^\circ \): The required value for \( f'_D \) is determined by drawing a -20dB/dec line from the 1-MHz point on the \( 20\log(1/\beta) = 20\log1 = 0 \text{dB} \) line. This line will intersect the 100-dB dc gain line at 10Hz.

Thus \( f'_D = 10 \text{Hz} = \frac{1}{2\pi(C_1 + C_C)R_1} \quad \therefore \quad C_C = 1 \mu\text{F} \)
Find the compensating capacitor \( C_f \) connected in the feedback path of the transistor.

By Eqs. (8.58) and (8.59):

\[
f'_{p1} \approx \frac{1}{2\pi g_m R_2 C_f R_1^2} \quad \text{and} \quad f'_{p2} \approx \frac{g_m C_f}{2\pi [C_1 C_2 + C_f (C_1 + C_2)]}
\]

Determine \( f_{p2} \): Assume \( C_f \gg C_2 \), then

\[
f'_{p2} \approx \frac{g_m}{2\pi (C_1 + C_2)} = 60.6 \text{ MHz}
\]

\( \Rightarrow \) The pole \( f'_{p2} \) moves to a frequency higher than \( f_{p3} \) (\( \approx 10 \text{ MHz} \)).

The new 2\(^{nd}\) pole is at \( f_{p3} \). This requires that the 1\(^{st}\) pole be located at 100Hz for \( \text{PM} \geq 45^\circ \):

\[
f'_{p1} = 100 \text{ Hz} = \frac{1}{2\pi g_m R_2 C_f R_1^2} \quad \therefore C_f = 78.5 \text{ pF}
\]

Conclusion: Using Miller compensation not only results in a much smaller compensating capacitor but, owing to pole splitting, also enables us to place the dominant pole a decade higher in frequency.
Homework #2
- Problems 42, 58, 61, 67, 70, 80